The use of a fibre anemometer in turbulent flows

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Quartz fibre anemometers have been used (as described in subsequent papers) to survey the velocity field of turbulent free convective air flows. This paper discusses the reasons for the choice of this instrument and provides the background information for its use in this way. Some practical points concerning fibre anemometers are mentioned. The rest of the paper is a theoretical study of the response of a fibre to a turbulent flow. An approximate representation of the force on the fibre due to the velocity field and the equation for a bending beam, representing the response to this force, form the basis of a consideration of the mean and fluctuating displacement of the fibre. Emphasis is placed on the behaviour when the spectrum of the turbulence is largely in frequencies low enough for the fibre to respond effectively instantaneously (as this corresponds to the practical situation). Incomplete correlation of the turbulence along the length of the fibre is taken into account. Brief mention is made to the theory of the higher-frequency (resonant) response in the context of an experimental check on the applicability of the low-frequency theory.

1. Introduction

The fibre anemometer (Schmidt & Beckmann 1930; Schmidt 1934; Kraus 1955, chapter E; Tritton 1959*b*) is a simple instrument that indicates fluid velocity by the deflexion of the free end of a fibre, commonly quartz, of which the other end is in effect clamped. The deflexion is observed through a tele-microscope with a calibrated eyepiece scale. Whilst its use in laminar flows has been successful, the fibre anemometer is at first sight an unpromising instrument for turbulent flows. However, there are situations in which the absence of any alternative has led to its use. It is the purpose of this paper to discuss some of the problems of such use and to present some information that facilitates it. Subsequent papers (Tritton 1963*a*, *b*) will describe experiments in which quartz-fibre anemometers were so used; this paper provides the background necessary for the interpretation of the results. The flows concerned are free convective ones. It is there that other methods of investigating turbulent flows run into difficulties (see § 2), and whilst most of the remarks in this paper are in principle generally applicable, their practical interest is probably confined to turbulent free convection.

Fibre anemometers have been used once previously in a turbulent flow—by Kraus (1940). However, he supposed, without critical discussion, that the mean fibre deflexion corresponded to the mean velocity. As will be seen below this calls for some caution.

2. Reasons for using fibre anemometer

The first choice of instrument in turbulent flow work is normally a hot-wire anemometer. There are a number of difficulties with this in free convection, each of which individually might be overcome but which cumulatively indicate the desirability of a different instrument. (i) In some free convective flows, velocity fluctuations are comparable with the mean velocity. A hot-wire anemometer is not good at discriminating between the two. (ii) At low speeds, King's law breaks down (Cooper & Linton 1934; Collis 1956), as a result of the interaction between forced and free convection from the wire. The speed at which this happens can be reduced below those often occurring in free convection only by using such a low wire temperature that point (iii) becomes particularly serious. (iii) The wire responds to both velocity and temperature variations and, although it is possible to allow for the latter, their existence produces loss of accuracy in the velocity measurements. (iv) A hot-wire anemometer changes its calibration slowly during use, largely as a result of its accumulating dust. In wind-tunnel work it is possible both to take precautions to minimize the dust and to recalibrate frequently without remounting the wire. In free convection neither of these is practicable.

In the set-up of Tritton (1963*a*) and Tritton (1963*b*) measurements close to the plate with a hot wire would have been further complicated by the heat losses to the plate (Wills 1962).

Contrastingly, the fibre anemometer discriminates well between mean and fluctuating velocities; can be used at lower speeds than are available for calibration (by use of a Reynolds number *vs* drag coefficient curve Tritton 1959*b*†); has only a small dependence, for which a correction may be made, on temperature; and retains its calibration much longer than a hot-wire anemometer. It is the first of these points that really decides in favour of a mechanical detector, having directional sensitivity, such as a fibre anemometer.

It is, of course, a severe disadvantage of the fibre anemometer that it does not give information in the form of an electrical signal. The consequent limitations are accentuated by the fact that averaging over rather long periods is sometimes required to give a true mean. There is undoubtedly a need for a new instrument capable of accurate quantitative velocity measurements in turbulent free convection. However, it is not at present clear how such an anemometer is to be designed, and it is likely that, if and when it is contrived, it will have to be used in conjunction with some other instrument, such as a fibre anemometer, that gives a direct qualitative impression of the characteristics of the velocity variations.

Meantime, we have to be content with such impressions and the limited quantitative data that can be obtained. The present state of our knowledge about free convection is such that this restricted information is not to be scorned. Even the qualitative impressions (indicating, for instance, whether there are bursts of high-frequency activity, how the fluctuations compare in magnitude with

† In addition to the information given there, there is relevant data in papers by White (1946) and Finn (1953), which I regrettably overlooked in 1959.

the mean velocity, and so on) require an understanding of the response of the anemometer to a turbulent flow field. §§ 4–7 discuss this matter of the response.

3. Some practical points

First, however, there are a few points of interest regarding the actual use of quartz-fibre anemometers.

The arrangement described in Tritton (1959*b*) of having the fibre cemented into the end of a piece of hypodermic tubing was found very satisfactory for the experiments of Tritton (1963*a, b*). Easy and successful mounting of a fibre depends rather critically on mixing the cement to just the right consistency. I used 'Polyfilla' (a commercial product intended for filling cracks in walls). A small amount was sucked into the end of the hypodermic tubing, and the end of the fibre gently pushed into this. The cement had to be just, but only just, sufficiently fluid for the fibre to be pushed in without any tendency to bend. It was found essential to allow at least 2 days for the cement to set. Even with this care it is possible that now and then a fibre may move in its mount, and to avoid spurious results it is wise to observe through the tele-microscope the behaviour of the fixed end whilst the fibre is intermittently subjected to an air flow considerably faster than that in which it is to be used.

It is necessary occasionally to clean fibres in constant use. Washing in ether has been found satisfactory; any very recalcitrant pieces of dust may be removed by gently running the tip of a finger along the fibre. Sometimes, a fibre picks up an electrostatic charge (with consequent rapid accumulation of dust); this is removed by leaving it near an α-particle source for a while.

One unexpected trouble has been encountered; fibres used for some while in turbulent flow develop a temperature-dependence in their zero reading (i.e. the position of the free-end in zero velocity), which is not to be observed in a newly made fibre. A likely explanation is that the continuous bending motion produces partial devitrification along one side of the fibre (it has been my practice to use a fibre repeatedly the same way round in the mean flow, so the necessary asymmetry exists†); the coefficients of linear expansion of fused and crystalline quartz are sufficiently different for this to produce appreciable bending when the fibre is heated. The onset of this trouble can easily occur unnoticed, so it is necessary to watch out for it.

For quantitative work the variation of Young's modulus of fused quartz fibre (with diameter or between samples) (Tritton 1959*a*) must be borne in mind. My procedure for the fibres used in the experiments of Tritton (1963*a, b*) was to use the resonant frequencies as an indication of the diameter, taking the Young's modulus from Tritton (1959*a*). This was satisfactory as the fibres were all nominally 20 or 45μ in diameter and the purpose of the measurements was to allow for deviations from these. In any new project, however, it would be necessary to make separate diameter measurements so that the resonant fre-

† If this explanation is correct, the devitrification occurs on the downstream side of the fibre; i.e. on the inside of the curve.

quencies could give the Young's modulus. This point applies, of course, only when the Reynolds number *vs* drag coefficient curve is being used to give a theoretical calibration curve, not when direct calibration is possible.

4. Response of a fibre anemometer to turbulence; introduction

Although one of the virtues of the fibre anemometer is that it gives a direct impression of the characteristics of a turbulent flow, the detailed relationship between the velocity field and the motion of the free end of the fibre is complex. The rest of this paper considers ways in which the theory of the response can help with the interpretation of the observations. We can then know better what limitations there are to the qualitative impressions obtained by watching a fibre (through, for example, a change in the length scale appearing the same as a change in intensity). The results may also facilitate quantitative use of the instrument.

First there are routine calculations on the fibre behaviour to be dealt with as cursorily as possible. The problem may be divided into the action of the velocity field in producing a force on the fibre and the response of the fibre to that force. The first will be dealt with by assuming that the force at any instant and any station on the fibre is related to the velocity then and there in the same way as in steady, two-dimensional flow. This is done largely as a matter of necessity, though the effect of unsteadiness can be regarded as a contribution to the damping of the fibre motion, and these points will be mentioned again in § 6.

Even with this assumption there is no general algebraic expression for the relationship between the velocity and the force. However, for the present purposes we may use as an approximation

$$F = AU + BU^2. \tag{1}$$

The principal justification of this is its simplicity; however, suitable choice of A and B can produce quite a close approximation to the actual curve. The choice must be made for the particular fibre by considering the Reynolds number range that the fluctuating velocity covers. (In general, the coefficients α and β in the non-dimensional form of (1),

$$C_D R^2 = \alpha R + \beta R^2, \tag{2}$$

respectively increase and decrease as the average R increases, though both sufficiently slowly for the ratio of the terms $\beta R/\alpha$ to increase.) Equation (1) does not make F an odd function of U, so it is intended for the case when there is a mean velocity in the $+U$ direction; however, occasional fluctuations to negative U should still be covered by the following analysis. In fact, it is also assumed that low velocities are involved; otherwise there is no advantage in constraining the parabola to go through the origin. These are the conditions in which a fibre anemometer is most likely to be used, but the modification of the analysis for different circumstances would be straightforward.

Dividing equation (1) into mean and fluctuating parts

$$\overline{F} + F' = A(\overline{U} + U') + B(\overline{U} + U')^2, \tag{3}$$

so that

$$\overline{F} = A\overline{U} + B\overline{U}^2 + B\overline{U'^2}. \tag{4}$$

For mean-velocity measurements in turbulent flow, it is important to be able to estimate a correction for the last term in (4). This requires a knowledge of $\overline{U'^2}$, which is one of the purposes of the following discussion of the fluctuations. Tritton (1963*a*) (in particular, the appendix) will provide an example.

Equation (3) gives

$$F' = (A+2B\overline{U})\,U' + B(U'^2 - \overline{U'^2}), \tag{5}$$

$$\overline{F'^2} = (A^2 + 4AB\overline{U} + 4B^2\overline{U}^2 - B^2\overline{U'^2})\,\overline{U'^2} + 2B(A+2B\overline{U})\,\overline{U'^3} + B^2\overline{U'^4}.$$

If we suppose that $\overline{U'^3} = 0$ and $\overline{U'^4} = 3(\overline{U'^2})^2$ (corresponding to a Gaussian probability distribution function)

$$\frac{\overline{F'^2}}{\overline{F}^2} = \frac{\overline{U'^2}}{\overline{U}^2}\,\frac{(A+2B\overline{U})^2 + 2B^2\overline{U'^2}}{\left[A + B\overline{U}\left(1+\dfrac{\overline{U'^2}}{\overline{U}^2}\right)\right]^2}. \tag{6}$$

This indicates that the curvature of the (U, F)-dependence increases the effect of the fluctuations relative to the mean behaviour.

For the theory of the response to the fluctuations, we have to use a linearized approximation to (5),

$$F' = (A+2B\overline{U})\,U'. \tag{7}$$

This is necessitated by the fact that the theory involves (as will be seen in § 5) correlations between the force fluctuations at different positions. Unless these are directly related to correlations of the velocity fluctuations the situation is complicated intolerably. Fortunately the curvature of the (U, F)-curves is sufficiently slight that (6) may be a reasonable approximation. It is certainly acceptable at larger values of $\overline{U'^2}/\overline{U}^2$ than the corresponding relationship for a hot-wire anemometer. The error in (7) is of the same order as would be produced by neglecting the last term in (4). The more accurate form is retained in the latter case as that part of the theory is to be applied in a more direct quantitative way.

The equation for the response of the fibre to the aerodynamic force will be taken as

$$m\frac{\partial^2 y}{\partial t^2} = F - EI\frac{\partial^4 y}{\partial x^4} - k\frac{\partial y}{\partial t}. \tag{8}$$

The damping term is probably not really linear, but, as this term will be neglected (see §§ 5 and 6) except for one special purpose in § 7, this point may be passed over in the present context. The boundary conditions are those of a beam clamped at one end, free at the other:

$$\left.\begin{aligned} y = \frac{\partial y}{\partial x} &= 0 \quad \text{at} \quad x = 0,\\ \frac{\partial^2 y}{\partial x^2} = \frac{\partial^3 y}{\partial x^3} &= 0 \quad \text{at} \quad x = l, \end{aligned}\right\} \tag{9}$$

and the quantity observed experimentally is

$$h = y \quad \text{at} \quad x = l.$$

Averaging† throughout equation (8) gives

$$\bar{F} = EI\frac{d^4\bar{y}}{dx^4}, \tag{10}$$

which may be integrated as usual to give

$$\bar{h} = \frac{\bar{F}l^4}{8EI}. \tag{11}$$

Hence, the only difficulty of using a measured value of $\bar{h}$, along with a laminar flow response curve, to indicate the mean velocity in a turbulent flow is the velocity fluctuation term in (4).

Investigating the response of the fibre to the fluctuating force on it is much the most complicated part of the problem. Basically, it consists of finding the statistical properties of y' and in particular h' resulting from the equation

$$m\frac{\partial^2 y'}{\partial t^2} = F' - EI\frac{\partial^4 y'}{\partial x^4} - k\frac{\partial y'}{\partial t} \tag{12}$$

(given by subtracting (10) from (8)), where F' is given by the turbulent field and on the basis of equation (7) is essentially U'.

The fibre has resonances and these will dominate the response unless little of the energy of the turbulence is in the appropriate frequencies. However, this exception is important; during the experiments to be described in Tritton (1963 *a, b*) the most vigorous motion of the fibre was clearly in frequencies lower than any of the resonances. Hence, the main consideration of the response (§ 5) will be formulated in a low-frequency approximation. This much simplifies matters, but calculations are still needed to allow for incomplete correlation of the velocity fluctuations along the length of the fibre.

The higher-frequency response, with the motion of the fibre dominated by its normal modes, forms a more-or-less separate study (required when inequality (14) in the next section is not satisfied). A theory of this motion has been developed along the lines of Powell's (1958) general theory of the response of structures to random forces, with a specification of the turbulence similar to that used by Liepmann (1955) in his study of the buffetting of aircraft. This theory will not be given in full here, as it plays no detailed role in the interpretation of fibre-anemometer observations.‡ However, a couple of results will be quoted in § 7, where they are required for one particular point.

5. Response of a fibre at low frequencies

For this case we neglect the inertia and damping terms in equation (12), i.e. we suppose that the time-scale of the turbulence is so large that for each point of the fibre at each instant there is effectively equilibrium between the aerodynamic and elastic forces.

† F' and y' are functions of both x and t, $\bar{y}$ is a function of x, whilst $\bar{F}$ is constant for uniform mean flow.

‡ Probably the main interest of this theory is in its application to the response of engineering structures subjected to turbulent flows.

The neglect of the fibre inertia requires that

$$\left|m\frac{\partial^2 y'}{\partial t^2}\right| \ll \left|EI\frac{\partial^4 y'}{\partial x^4}\right|, \tag{13}$$

which will be fulfilled if most of the energy of the turbulence is in frequencies satisfying

$$\nu^2 \ll EI/ml^4. \tag{14}$$

The lowest resonant frequency of a fibre is given by

$$\nu_1^2 = 0{\cdot}313EI/ml^4.$$

Thus, it should be a satisfactory approximation to neglect the inertia term when the stimulation of resonances is not involved. Clearly, for this to be an advantage we need the damping term to be small too. The permissibility of this will be discussed in § 6.

Equation (12) now reduces to the 'static' problem

$$EI\frac{\partial^4 y'}{\partial x^4} = F'(x). \tag{15}$$

The boundary conditions (9) apply to y' as well as y, and so, by summing the displacements produced by the forces at different values of x

$$h' = \frac{1}{EI}\int_0^l F'(x)\,(\tfrac{1}{2}x^2 l - \tfrac{1}{6}x^3)\,dx. \tag{16}$$

Hence, the observed mean-square displacement is

$$\overline{h'^2} = \frac{1}{E^2I^2}\int_0^l\int_0^l \overline{F'(x_1)\,F'(x_2)}\,(\tfrac{1}{2}x_1^2 l - \tfrac{1}{6}x_1^3)\,(\tfrac{1}{2}x_2^2 l - \tfrac{1}{6}x_2^3)\,dx_1\,dx_2. \tag{17}$$

Writing $X = x_2 - x_1$ and $\overline{F'(x_1)F'(x_2)} = \overline{F'^2}R(X)$ (since we are considering $\overline{U}$ to be the same all along the fibre, equation (7) implies that $R(X)$ may be taken as the velocity correlation coefficient of the turbulence),

$$\overline{h'^2} = \frac{\overline{F'^2}}{E^2I^2}\int_0^l (\tfrac{1}{2}x_1^2 l - \tfrac{1}{6}x_1^3)\left\{\int_{-x_1}^{l-x_1} [\tfrac{1}{2}(X+x_1)^2 l - \tfrac{1}{6}(X+x_1)^3]\,R(X)\,dX\right\}dx_1. \tag{18}$$

Although $R(X)$ is really one of the quantities that should be measured in a survey of a turbulent flow, there is sufficient information available (Mickelsen 1955; Townsend 1956; Grant 1958; Hinze 1959; Comte-Bellot 1961) to indicate the general shapes of $R(X)$ curves that occur. It thus seems useful to have on record evaluations of (18) for a few cases that represent the range of behaviours of $R(X)$. This is the main purpose of this section.

The representative cases chosen are:

(i) $R(X) = \exp(-|X|/a)$,

(ii) $R(X) = \exp(-X^2/a^2)$,

(iii) $R(X) = (1+0{\cdot}2214\,|X|/a)^{-5}$,

(iv) $R(X) = \exp(-0{\cdot}573\,|X|/a)\cos(\tfrac{3}{2}\,0{\cdot}573X/a)$,

(v) $R(X) = 4{\cdot}5\exp(-0{\cdot}779\,|X|/a) - 3{\cdot}5(1+\tfrac{1}{5}\,0{\cdot}779\,|X|/a)^{-5}$.

The constants multiplying $|X|/a$ are chosen so that in each case a is defined by $R(a) = e^{-1}$.

The first three cases indicate how much the shape of the correlation curve may affect the result even when $R(X)$ is positive throughout. Case (i) is a shape that, by appropriate choice of a, approximates well to many such correlation functions. Others may be, for the most part, intermediate between cases (i) and (ii). At $X = 0$, $R(X)$ must behave more like case (ii) than case (i), but commonly only for such a short distance that this region is not detectable. Batchelor & Proudman (1956) have pointed out that the asymptotic variation of correlations at

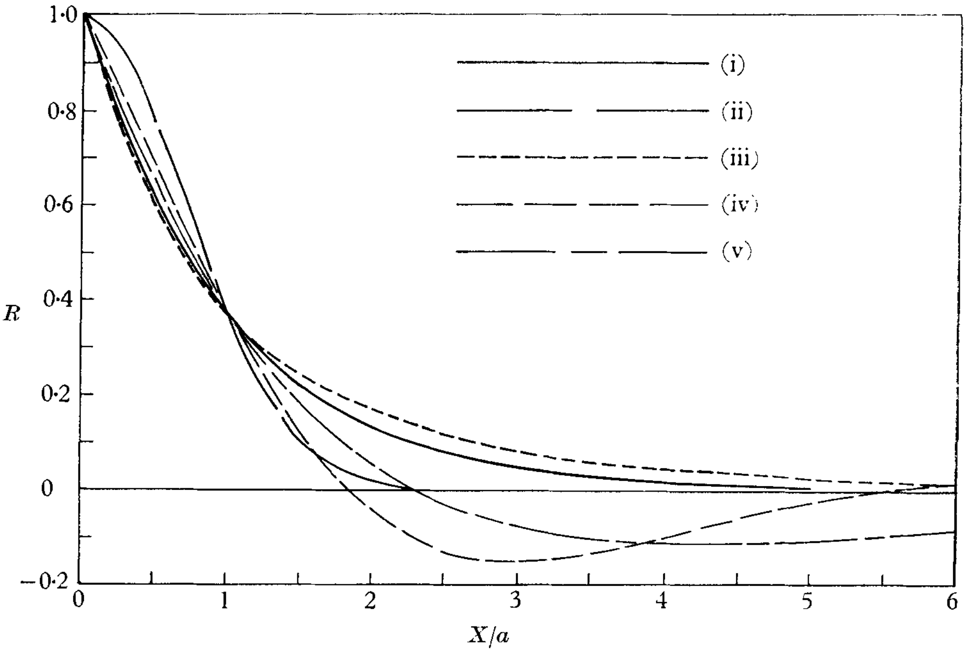

FIGURE 1. The five representative correlation functions.

large X is liable to be proportional to X^{-n}, with n around 5, rather than exponential. Case (iii) is included as an indication of how much difference this might make.

We are concerned with correlations of velocity components normal to the separation, and $R(X)$ is likely (though not certain) to have a negative region. Cases (iv) and (v) cover this eventuality. The particular functions were chosen partly because the results could be calculated from those of the earlier cases. However, they can again show the effect of different detailed behaviours, particularly at large X and the five cases should indicate the range of possibilities. The constants in cases (iv) and (v) are chosen so that the heights of the negative parts of R are around the largest observed. The heights are, respectively, about 0·15 and 0·11, but case (iv) has further positive regions at larger X, where case (v) remains negative.

Figure 1 shows the five correlation curves. Figure 2 shows the corresponding plots of $(\overline{h'^2})^{\frac{1}{2}}\, 8EI/(\overline{F'^2})^{\frac{1}{2}}\, l^4$ against a/l to a logarithmic scale. This ordinate is chosen as it must tend to 1 as $a/l \to \infty$ (complete correlation). The choice of a, the distance in which the correlation falls to $1/e$, as the length scale of each case is obviously somewhat arbitrary. For cases (iv) and (v), rather larger length scales (relative to the other cases) might be preferred particularly if the large

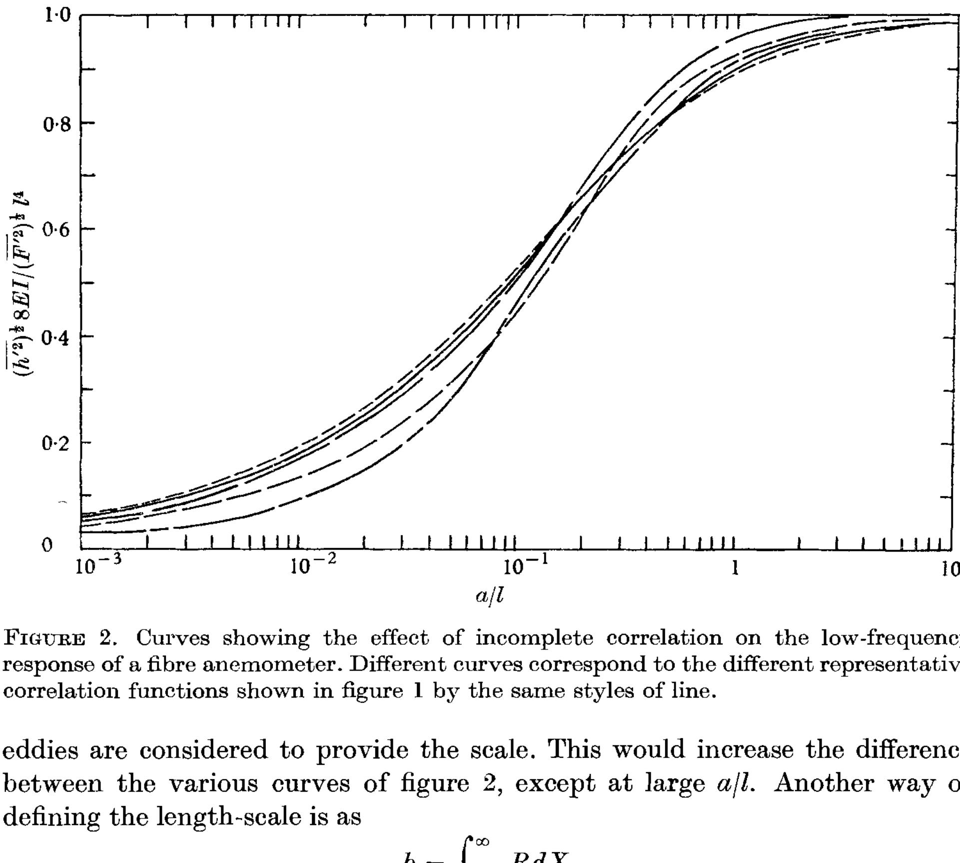

FIGURE 2. Curves showing the effect of incomplete correlation on the low-frequency response of a fibre anemometer. Different curves correspond to the different representative correlation functions shown in figure 1 by the same styles of line.

eddies are considered to provide the scale. This would increase the difference between the various curves of figure 2, except at large a/l. Another way of defining the length-scale is as

$$b = \int_{-\infty}^{\infty} R\,dX.$$

It can be shown that the use of this instead of a would necessarily make the curves of figure 2 coincide at small b/l; asymptotically

$$\frac{(\overline{h'^2})^{\frac{1}{2}}\, 8EI}{(\overline{F'^2})^{\frac{1}{2}}\, l^4} = 1{\cdot}83(b/l)^{\frac{1}{2}}.$$

For the five cases, (i) $b = 2a$; (ii) $b = 1{\cdot}77a$; (iii) $b = 2{\cdot}26a$; (iv) $b = 1{\cdot}07a$; (v) $b = 0{\cdot}321a$. By considering the curves of figure 2 shifted accordingly, one sees that the improvement (in the sense of making the curves closer to one another) is confined to low b/l; elsewhere there is a deterioration. Also, although b is a convenient scale when R is positive throughout, its physical significance is doubtful when there is a negative region.

The most useful outcome of all this is that the curves in figure 2 for the different cases form a fairly narrow band. Hence, it will not always be necessary to know the detailed behaviour of the correlation; some discussion is useful in terms of the average curve. Moreover, the abscissa of figure 2 being logarithmic, order-of-magnitude knowledge of a/l may be sufficient. The details of any such treatment will depend on its purpose. An example is provided by the considerations in the appendix to the following paper (Tritton 1963*a*). a/l must not be too small (not less than 0·03, say) for this to work well. Otherwise, the fractional differences are appreciable and an endeavour should be made to replace a/l by b/l.

6. Damping

We now examine whether the neglect in § 5 of the damping term of equation (12) was allowable.

There are four sources of damping: (i) internal friction; (ii) the drag on the fibre as it moves through the air; i.e. the effect of the velocity of the air relative to the fibre being not U but $U - \partial y/\partial t$; (iii) the motion of the fibre causing the velocity measured to be that at a varying position; (iv) the inertia of the air; if the velocity fluctuations occur too rapidly the flow past the cylinder, and so the drag, may not settle down to that corresponding to the instantaneous velocity before a further change has occurred.

The information about the properties of fused quartz needed for an estimate of (i) does not exist, so far as I can ascertain. However, it is reasonable to hope that it is small and to consider only the three aerodynamic effects. Some justification for this is provided by the width of the resonances observed in the experiments (Tritton 1959*a, b*) with fibres in front of a loudspeaker. These are of the same order as would be produced by effect (ii), effects (iii) and (iv) being absent in this arrangement.

It is straightforward to find criteria for each of the three aerodynamic effects to be negligible in the low-frequency problem of § 5. For (ii), the requirement is $U' \gg \partial y'/\partial t$, or, taking the extreme case of the free end, $U' \gg \partial h'/\partial t$. Now

$$\frac{\partial h'}{\partial t} = \frac{1}{EI}\int_0^l \frac{\partial F'}{\partial t}\left(\tfrac{1}{2}x^2 l - \tfrac{1}{6}x^3\right)dx,$$

$$\sim \frac{l^4}{8EI}\nu F' \sim \frac{l^4}{8EI}\nu(A + 2B\overline{U})\,U',$$

where ν is the reciprocal of the time-scale of the turbulence. The requirement is thus

$$\frac{l^4}{8EI}(A + 2B\overline{U})\,\nu \ll 1. \tag{19}$$

Because of effect (iii), the velocity producing the drag is that at y and not that at the fixed position $\overline{y}$.

$$\begin{aligned} U(y, t=0) &\simeq U(\overline{y}, -y'/U) \\ &\simeq U(\overline{y}, 0) - \frac{y'}{U}\frac{\partial U}{\partial t}. \end{aligned} \tag{20}$$

The correction is as large as $\partial y/\partial t$ only when U' is comparable with U. Hence, it need not be considered when (ii) is not.

Regarding (iv), a simple consideration of the Navier–Stokes equation indicates that the time-scale with which a deviation from the steady state tends to that state is $T \sim L/U$, where L is the length scale and U is the *total* velocity scale. Calculations by Payne (1958) of the drag produced by a starting flow past a cylinder confirm this if L is taken as the diameter. Hence, the requirement for this source of damping to be negligible is

$$\nu \ll U/L. \tag{21}$$

(We may note in passing that (21) amounts to requiring the length scale of the turbulence to be large compared with L. When this is fulfilled, another approximation mentioned in § 4, that the force at a point of the fibre is not influenced by the instantaneous variations of the velocity along the fibre's length, is also satisfactory.)

Neither (19) nor (21) is necessarily fulfilled, but inserting values of the various quantities, and remembering that ν is small compared with the resonant frequencies whenever the theory is useful, indicates that they were fulfilled for the fibres I have used. Indeed, it seems likely that they would be fulfilled for any quartz-fibre anemometer of practical value.

7. Use of the results in studying turbulent flows

The purpose of the previous sections has been to provide a general picture of the behaviour of a fibre anemometer as a background to its use for investigating turbulent flows. Rather than generalizing about the ways in which the foregoing results can be so used, it seems more useful to allow the following papers (Tritton 1963 *a*, *b*) to provide examples.

Often the practical use of the theory may be qualitative and not very specific. The purpose of this paper has been as much to give a general understanding of fibre behaviour for such qualitative use as to provide quantitative results. For example, if a flow has two modes (such as the 'quiescent' and 'active' periods observed by Townsend 1959 above a heated horizontal surface), then it is useful to know how far an increase in fibre amplitude implies an increase in intensity or how large a change in length-scale would be needed to explain it without an increase in intensity. Figure 2 helps with this.

When the low-frequency theory of § 5 is being used in the assessment of observations, it is necessary to have some check of its validity. In other words, we may not know initially whether inequality (14) is being fulfilled. The question then arises whether non-fulfilment would be adequately revealed by the stimulation of resonances. The remainder of this section is the answer to this question.

We compare the low-frequency response and the first resonant response for an imaginary turbulent motion in which the energy is uniformly distributed over all frequencies up to some cut-off, Ω, rather greater than ω_1 (the frequency of the first resonance). Then, from equations (7) and (18),

$$(\overline{h'^2})_{\text{low freq.}} = (A + 2B\overline{U})^2\, \overline{U'^2}\, \frac{l^8}{64E^2I^2}\lambda_1, \tag{22}$$

where λ_1 is a non-dimensional scaling factor to allow for incomplete correlation, being the ratio of the value of the double integral in (17) to its value when correlation is complete. From the theory of the resonant response mentioned in § 4, it may be shown that

$$(\overline{h'^2})_{\text{first res.}} = \frac{\pi}{2km\omega_1^2}\eta_1^2(l)\,(A+2B\overline{U})^2\frac{\overline{U'^2}}{\Omega}\lambda_2\left[\int_0^l \eta_1(x)\,dx\right]^2$$

$$= 0{\cdot}0904\frac{l^4\overline{U'^2}(A+2B\overline{U})^2\lambda_2}{kEI\Omega}, \qquad (23)$$

where m, k, and EI are the quantities appearing in equation (8), η_1 is the eigenfunction of the first normal mode, normalized to

$$\int_0^l \eta_1^2\,dx = 1,$$

and λ_2 plays the role in equation (23) corresponding to that of λ_1 in (22). On the reasonable assumption that the second of the effects listed in § 6 is the main source of damping,

$$k = A+2B\overline{U},$$

and so

$$\frac{(\overline{h'^2})_{\text{first res.}}}{(\overline{h'^2})_{\text{low freq.}}} \sim \frac{6EI}{l^4\Omega(A+2B\overline{U})}\frac{\lambda_2}{\lambda_1}.$$

Since $AU+BU^2 \sim F \sim 8EIh/l^4$ (U, F, and h here denoting values for which the fibre is typically used) and $\Omega \sim \omega_1$, the square root of this ratio is of the order of magnitude

$$\left(\frac{U}{h\omega_1}\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{2}}.$$

It might be thought that λ_2/λ_1 should be taken fairly small to allow for the fact that the higher-frequency motions producing $(\overline{h'^2})_{\text{first res.}}$ may be correlated over smaller distances than the lower-frequency ones producing $(\overline{h'^2})_{\text{low freq.}}$. An important inference of the theory of the resonant response mentioned in § 4 is that this effect is insignificant. Thus $\lambda_2/\lambda_1 \sim 1$ and

$$\left\{\frac{(\overline{h'^2})_{\text{first res.}}}{(\overline{h'^2})_{\text{low freq.}}}\right\}^{\frac{1}{2}} \sim \left(\frac{U}{h\omega_1}\right)^{\frac{1}{2}}.$$

In practice this has usually been in the range 1 to 5. Hence, the stimulation of the first resonance is vigorous enough to reveal when a fibre ought not to be used on the assumption of zero fibre inertia. The consequent motion is usually quite distinctive, but in cases of doubt can be detected with the aid of a stroboscope.

Much of this work was done when I was working in the Cavendish Laboratory, Cambridge, and described in Tritton (1960) (where the results are presented in greater detail). Since coming to Bangalore, I have extended the work, and I am grateful to Mr S. Durvasula for some helpful discussions.

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